Collective Quantum Dot Inversion and Amplification of Photon and Phonon Waves

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The possibility of steady-state population inversion in a small sample of strongly driven two-level emitters like quantum dots (QDs) in micro-cavities, and its utilization towards amplification of light and acoustic waves is investigated theoretically. We find that inversion and absorption spectrum of photons, and phonons crucially depend on the interplay between the intrinsic vacuum and phonon environments. The absorption profiles of photons and phonons show marked novel features like gain instead of transparency and absorption reversed to gain, respectively. Furthermore, we report collectivity induced substantial enhancement of inversion and pronounced gain in the photon, and phonon absorption spectrum for a wavelength size QD ensemble.

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Even after fifty years of the advent of laser, achieving population inversion and lasing in novel systems remain a topic of continuing interest [1]. In particular, inversion in solid-state emitters like quantum dots (QDs) has attracted considerable interests recently [2–4], due to the possibility of lasing in a driven two-level system [5, 6] contrary to the common notion of multilevel manifolds [1]. Semiconductor QDs are novel nano-structures [7] with unique engineerable features like large optical dipole moments and emission wavelengths. They can also be integrated efficient to micro-cavities and waveguides [8]. Their similarities to two-level atoms have been demonstrated in phenomena like Autler-Townes doublet [9], Mollow triplet in resonance fluorescence [10, 11], spectral line narrowing [12–14] and superradiance [15]. Furthermore, collectivity induced fluorescence and decoherence of a few nearly identical QDs have also been investigated [16]. Additonally, due to potential application as solid state qubits numerous coherent optical studies of this system has been undertaken in the past decade [17–21]. During last few years, sensitivity of QDs to the nature of their phonon environment has been exploited in numerous investigations. Phonon-assisted damping of Rabi oscillations [22–24], effect of phonons on polarization-entangled photons [25], phonon induced transitions of excitons to cavity photons [26], heat pumping with optically driven excitons in phonon environment [27] and effect of electron-phonon coupling on resonance fluorescence spectrum [28–30] have been reported. Furthermore, phonon mediated population inversion in a single QD-cavity system driven by a continuous wave [31] and a pulse laser excitation [32] was proposed recently.

In light of these works, here, we propose a novel scheme to achieve and enhance population inversion in an ensemble of strongly driven two-level solid-state emitters like QDs via coupling to a phonon-photon reservoir. We find important interplay of the phonon-photon reservoirs on the QD-dynamics and in creation of steady-state inversion in both bare- and dressed-states. As a key finding, we report phonon induced unique features in the QD photon absorption spectrum like, gain instead of transparency at resonance and absorption switched to gain for bare-state inversion. We also find gain in the phonon absorption spectrum under condition of dressed-state inversion which can be harnessed for amplification of phonon waves at THz frequencies. Additionally, for an wavelength size ensemble of QDs, we report collectivity assisted enhancement of inversion and improvement of gain in the photon or phonon absorption spectra.

Model: We consider an ensemble of QDs embedded in a substrate (such as InGaAs encased in GaAs substrate) or in a micro-cavity (like that formed by InAs/GaAs Bragg reflectors) illuminated by a laser of frequency ω_L. As shown schematically in Fig. (1) the ground state $|1\rangle$ and the state containing a trapped electron-hole pair (exciton) $|2\rangle$ in each QD form a two-level system with transition frequency ω_x. The linear dimension of the ensemble is smaller or of the order of the relevant emission wavelength. The laser is detuned from the exciton transition by $\Delta = \omega_x - \omega_L$ and drives it with a Rabi frequency of Ω. Furthermore, the QDs interact with the solid state environment (phonon reservoir) and with field modes of the electromagnetic vacuum (photon reservoir) leading to different incoherent decay processes.

The Hamiltonian describing the interaction of N QDs with the laser and the vacuum and phonon reservoirs in a frame rotating with the laser frequency ω_L can be represented as:

$$\begin{aligned}\mathcal{H} &= \sum_k \hbar\delta_k a_k^\dagger a_k + \sum_p \hbar\omega_p b_p^\dagger b_p + \hbar\sum_{j=1}^{N}\Delta S_j^z \\ &+ \hbar\sum_{j=1}^{N}\Omega\,(S_j^+ + S_j^-) + \Big\{ i\sum_{j=1}^{N}\Big[\sum_p \lambda_p S_j^+ S_j^- b_p^\dagger \\ &+ \sum_k (\vec{g}_k\cdot\vec{\wp}_j) a_k^\dagger S_j^-\Big] + H.c.\Big\}. \qquad (1)\end{aligned}$$

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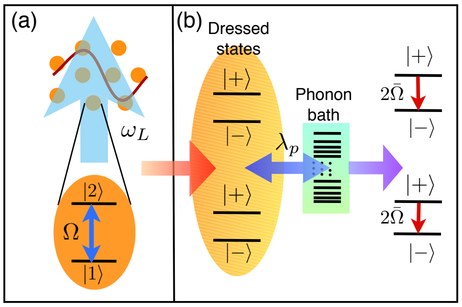

FIG. 1: (color online) Schematics of interaction of a collection of QDs with a coherent field and phonon reservoir. (a) QDs as two-level systems embedded in a substrate or in a micro-cavity of emission wavelength size, driven by a laser of Rabi frequency Ω. (b) In strong field, the QDs are dressed by the laser giving rise to dressed-states $|\pm\rangle$ and characteristic Mollow triplet in emitted photon radiation. However in presence of phonons, additional transitions at the Rabi frequency $2\bar{\Omega}$ among the dressed-states are induced.

In Eq. (1) the first three terms represent the free Hamiltonians of the electromagnetic vacuum, the phonon reservoir and the two-level QDs. The fourth term corresponds to the laser-QD interaction, while the two terms inside the curly brackets describe, respectively, the QDs interaction with the surrounding phonon and vacuum reservoirs. Both the vacuum and phonon reservoirs are represented in the form of harmonic oscillators with $a_k^\dagger(a_k)$ and $b_p^\dagger(b_p)$ as the creation (annihilation) operator of the vacuum field in mode $\vec{k}$ and phonons in mode $\vec{p}$ having frequencies ω_k and ω_p respectively. They satisfy the standard boson commutation relations: $[Q_u, Q_{u'}^\dagger] = \delta_{uu'}$ and $[Q_u, Q_{u'}] = [Q_u^\dagger, Q_{u'}^\dagger] = 0$, $(Q = \{a, b\}, u = \{k, p\})$. The laser is detuned to the vacuum mode $\vec{k}$ by $\delta_k = \omega_k - \omega_L$. The terms λ_p, $\vec{g}_k$ and $\vec{\wp}_j$ are respectively the QD-phonon coupling constant, the mode function of the three-dimensional multi-mode field [33] and the dipole moment of the j-th QD. The raising and lowering operators for each QD are denoted by $S_j^+ = |2\rangle_{jj}\langle 1|$ and $S_j^- = (S_j^+)^\dagger$, while $S_j^z = (|2\rangle_{jj}\langle 2| - |1\rangle_{jj}\langle 1|)/2$, obeying the standard su(2) angular momentum commutation relations.

In the following we consider the QDs system driven by an intense laser. This leads to the dressing of both the bare-states in each QD creating the dressed-states, $|+\rangle_j = \sin\theta|1\rangle_j + \cos\theta|2\rangle_j, |-\rangle_j = \cos\theta|1\rangle_j - \sin\theta|2\rangle_j$ where $\tan 2\theta = (2\Omega/\Delta)$. Hence, it is imperative to continue the further analysis in the dressed-state basis. To study the collective quantum dynamics of the strongly driven QD ensemble interacting with vacuum and phonon reservoirs, we next derive a dressed basis master equation (DME) in the interaction picture.

Collective Quantum Dynamics of QDs: The general form of the master equation in the interaction picture is given by [33]: $\partial\rho_{sb}/\partial t = 1/i\hbar\,[\mathcal{H}_I(t), \rho_{sb}(0)] - 1/\hbar^2 \int_0^t dt'\,[\mathcal{H}_I(t), [\mathcal{H}_I(t'), \rho_{sb}(t')]]$. Here, ρ_{sb} is the density operator of the combined QDs + reservoirs (both vacuum and phonons) in the interaction picture. The dressed-state interaction Hamiltonian $\mathcal{H}_I$ can be evaluated by using the dressed-state transformations in Eq (1) and an unitary transformation by the operator $e^{(i/\hbar)H_0 t}$, where $H_0 = \sum_k \hbar\delta_k a_k^\dagger a_k + \sum_p \hbar\omega_p b_p^\dagger b_p + \hbar\bar{\Omega}\sum_{j=1}^N R_j^z$. On substituting the interaction Hamiltonian and taking trace over the vacuum field and phonon modes, after some tedious algebra, we get the DME for the reduced density operator ρ of the QDs in the Born-Markov approximation, and weak coupling regime as:

$$\frac{\partial\rho}{\partial t} + i\tilde{\Omega}\,[R^z, \rho] = -\chi_0 \sum_{l,j} [R_l^z, R_j^z\rho]$$
$$-\chi_+ \sum_{l,j} [R_l^+, R_j^-\rho] - \chi_- \sum_{l,j} [R_l^-, R_j^+\rho] + H.c., \quad (2)$$

where $\tilde{\Omega} = \bar{\Omega} - \Delta_p$ with $\bar{\Omega} = \sqrt{(\Delta/2)^2 + \Omega^2}$ being the generalized Rabi frequency, while Δ_p is an additional shift due to QD-phonon coupling. The well known Lamb shift due to vacuum fields has been absorbed in the exciton frequency. The dressed basis operators are defined as: $R_j^z = |+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -|$ and $R_j^+ = |+\rangle_{jj}\langle -|, R_j^- = (R_j^+)^\dagger$ obeying the commutator relation $[R_{\alpha,\beta}^{(j)}, R_{\alpha',\beta'}^{(l)}] = \delta_{jl}\{\delta_{\beta\alpha'}R_{\alpha\beta'}^{(j)} - \delta_{\beta'\alpha}R_{\alpha'\beta}^{(j)}\}$.
In Eq. (2) $\chi_{0,\pm}$ are the renormalized decay rates given by: $\chi_0 = \left(\Gamma_0 \sin^2 2\theta + \Gamma_d \cos^2 2\theta\right)/4$, $\chi_+ = [(\bar{n}+1)\Gamma_p + \Gamma_d]\sin^2 2\theta/4 + \Gamma_+ \cos^4\theta$ and $\chi_- = (\bar{n}\Gamma_p + \Gamma_d)\sin^2 2\theta/4 + \Gamma_- \sin^4\theta$. Here, $\Gamma_0 = (\mathcal{G}/\hbar)^2\kappa/[\kappa^2 + \delta_c^2]$, $\Gamma_\pm = (\mathcal{G}/\hbar)^2\kappa/[\kappa^2 + (\delta_c \mp 2\bar{\Omega})^2]$, and $\Gamma_p \equiv \Gamma_p(2\bar{\Omega}) = \pi\sum_p (\lambda_p/\hbar)^2\,\delta(\omega_p - 2\bar{\Omega})$. The term $\mathcal{G} = \vec{g}\cdot\vec{\wp}$ is the vacuum-QD coupling constant while $\sin 2\theta = \Omega/\bar{\Omega}$, $\cos^2\theta = (\bar{\Omega} + \Delta/2)/2\bar{\Omega}$, and $\sin^2\theta = (\bar{\Omega} - \Delta/2)/2\bar{\Omega}$. Further, $\bar{n} \equiv \bar{n}(2\bar{\Omega}) = [\exp\left(2\hbar\bar{\Omega}/k_B T\right) - 1]^{-1}$ is the mean phonon number at frequency $2\bar{\Omega}$ and temperature T, with k_B being the Bolzmann constant. The simplified expressions for $\Gamma_{0,\pm}$ is derived under the assumption that the QDs interact with a broad-band cavity mode of frequency ω_c and decay rate κ. The driving laser is detuned to the cavity frequency by $\delta_c = \omega_c - \omega_L$. Note that the correlation functions for the cavity vacuum field and phonon operators are taken in the standard form [33]. The decay terms included in the renormalized decay rates can be physically explained as follows: Γ_0 and $\Gamma_\pm$ are the radiative decay rates of the QDs dressed transitions involving the upper and lower doublets $|\pm\rangle \to |\pm\rangle$ and $|\pm\rangle \to |\mp\rangle$ ($|2\rangle \to |1\rangle$), respectively. Γ_p is the phonon induced dissipation rate of the QDs at the generalized Rabi frequency $2\bar{\Omega}$ between the dressed-state doublets of $|2\rangle$ as well as $|1\rangle$ (see Fig. 1b) and Γ_d is the pure dephasing rate of QDs due to phenomenon like elastic phonon scattering [25]. Note that in deriving Eq. (2)

in the standard Lindbald form, we have performed the secular approximation, *i.e.* we neglected the terms rotating with the frequency $2\bar{\Omega}$ and higher meaning that $2\bar{\Omega} \gg N\Gamma(1+\bar{n})$. Here Γ^{-1} typically corresponds to the smallest time scale of the system. The corrections to the results obtained in the secular approximation are of the order of $[N\Gamma(1+\bar{n})/\Omega]^2$ and can be neglected in the intense field limit.

Fluorescence and absorption spectrum: To understand the underlying physics of the QD dynamics, we next investigate the steady-state fluorescence and absorption spectrum of a single QD. The total steady-state photon fluorescence spectrum defined in the standard form [33], under secular approximation and in the dressed-state basis is evaluated as: $S_t(\omega) = S_c(\omega) + S(\omega)$, where $S_c(\omega)$ and $S(\omega)$ corresponds to the coherent and incoherent parts and are given by:

$$S_c(\omega) = \frac{\pi}{4}\langle R^z\rangle_{ss}^2 \sin^2 2\theta \delta(\omega-\omega_L), \tag{3}$$

$$\begin{aligned} S(\omega) &= \sin^4\theta\left(\frac{\chi_+}{\chi_+ + \chi_-}\right)\frac{\Gamma_{ML}}{\Gamma_{ML}^2 + (\omega-\omega_L+2\tilde{\Omega})^2} \\ &+ \sin^2 2\theta \frac{\chi_+\chi_-}{(\chi_+ + \chi_-)^2}\frac{\Gamma_{MC}}{\Gamma_{MC}^2 + (\omega-\omega_L)^2} \\ &+ \cos^4\theta\left(\frac{\chi_-}{\chi_+ + \chi_-}\right)\frac{\Gamma_{MR}}{\Gamma_{MR}^2 + (\omega-\omega_L-2\tilde{\Omega})^2}. \end{aligned} \tag{4}$$

The corresponding dressed-state absorption spectrum [34] in the secular approximation, for photons is :

$$\begin{aligned} A(\nu) &= i\langle R^z\rangle_{ss}\Bigg\{\sin^4\theta\frac{\Gamma_{ML} + i(\nu-\omega_L+2\tilde{\Omega})}{\Gamma_{ML}^2 + (\nu-\omega_L+2\tilde{\Omega})^2} \\ &- \cos^4\theta\frac{\Gamma_{MR} + i(\nu-\omega_L-2\tilde{\Omega})}{\Gamma_{MR}^2 + (\nu-\omega_L-2\tilde{\Omega})^2}\Bigg\}, \end{aligned} \tag{5}$$

while for phonons is given by:

$$A(\nu_p) = -\frac{i}{4}\langle R^z\rangle_{ss}\sin^2 2\theta\Bigg\{\frac{\Gamma_{ML} + i(\nu_p - 2\tilde{\Omega})}{\Gamma_{ML}^2 + (\nu_p - 2\tilde{\Omega})^2}\Bigg\}. \tag{6}$$

Here, $\langle R^z\rangle_{ss}$ is the single QD dressed basis steady-state population inversion given by: $(\chi_- - \chi_+)/(\chi_- + \chi_+)$. $\Gamma_{ML} = \Gamma_{RL} = 4\chi_0 + \chi_+ + \chi_-$ and $\Gamma_{MC} = 2(\chi_+ + \chi_-)$ with ν, ν_p being the weak photon and phonon probe frequencies, respectively. In Fig. 2(a) & 2(b) we plot Eq. (4) and imaginary part of Eq. (5). The solid and dashed lines in the plot corresponds to resonant and non-resonant coherent excitation of the QD. We have assumed bad cavity limit *i.e.* $\kappa \gg \mathcal{G}, \lambda_p$ and approximated $\Gamma_{0,\pm} = \gamma$, with γ being the QD radiative decay rate in the cavity. The photon fluorescence spectrum shown in Fig. 2(a) is similar to the Mollow spectrum well known for atoms, however, with certain key modifications due to the phonon bath. The position of the Mollow sidebands are shifted to $\pm 2\tilde{\Omega}$ with phonon induced broadening of their widths Γ_{ML} and that of the central line Γ_{MC} as can be easily seen from their respective analytical forms. Additionally, in presence of phonons the sidebands are asymmetric with the ratio of the right to left peak heights as: $[(2\bar{\Omega}+\Delta)/(2\bar{\Omega}-\Delta)]^2\chi_-/\chi_+$. This asymmetry is pronounced for non-resonant driving of the QD as shown in Fig. 2(a) by the dashed curve. For resonant drive however, the asymmetry can be prominent in the conditions: $\bar{n} \ll 1$ (low temperature reservoir) and $\Gamma_p \gg (\Gamma_d, \gamma)$. The coherent part of the fluorescence spectrum $S_c(\omega)$, that signifies the elastic scattering process in presence of phonons, does not vanish even at resonance and has an amplitude $\propto \Gamma_p^2/[(2\bar{n}+1)\Gamma_p + 2(\Gamma_d+\gamma)]^2$. Note that, such phonon induced modifications of the Mollow triplet has been observed in a recent experiment [30].

The imaginary part of the photon absorption spectrum shown in Fig. 2(b) is strikingly different from a typical Mollow absorption spectrum of a two-level atom. As a key finding, for resonant excitation of a QD we report *phonon induced gain and absorption* in the spectrum at $\nu - \omega_L = \mp 2\tilde{\Omega}$ respectively with peak heights $\propto \langle R^z\rangle_{ss}/4$. Note that, these characteristics are absent in atomic system, where there is no phonon coupling and the probe field is transparent at resonance as $\langle R^z\rangle_{ss} = 0$. Furthermore, for both resonant and non-resonant excitation we find from Eq. (5) broadening of the gain and absorption lines due to phonons. The dispersion characteristics of the absorption spectrum given by the real part of Eq. (5) behave in accordance with the absorption characteristics and have no additional features.

The imaginary part of Eq. (6) exhibits another remarkable feature, *gain* in the phonon absorption spectrum for *population inversion in the dressed-state.* This gain can, in principle, lead to amplification of a weak phonon wave in presence of an additional phonon cavity [35, 36] tuned to the dressed transition at the shifted generalized Rabi frequency of $2\tilde{\Omega}$ (see Fig. 1b). Note that the phonon gain profile is enhanced only in presence of the vacuum reservoir as can be seen from the expressions for $\chi_{0,\pm}$.

For a collection of N QDs in an ensemble of the size of emission wavelength, the emission-absorption spectrum is modified due to collective effects. The spectral lines are broadened and enhanced, with the enhancement proportional to N^2 and N, respectively, for the Mollow peaks in photon fluorescence and gain profile in the photon or phonon absorption [34]. In the following we discuss another key finding of the paper: *collectivity assisted enhancement* of steady-state population inversion.

Population inversion in QDs: In order to study the collective steady-state population inversion of the QDs in bare and dressed basis we need to obtain the steady-state solution of Eq. (2). For this purpose we first define the dressed-state collective operators as: $\mathcal{R}^{\pm} = \sum_{j=1}^{N} R_j^{\pm}, \mathcal{R}^z = \sum_{j=1}^{N} R_j^z$ and then look for a solution of the form [6]

$$\rho_{ss} = Z^{-1}\exp[-\eta\mathcal{R}^z], \tag{7}$$

where the normalization Z is determined by the requirement $\mathsf{Tr}\{\rho_s\} = 1$. The unknown variable η can be eval-

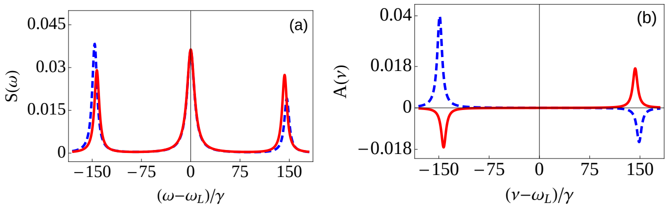

FIG. 2: (color online) The fluorescence and absorption spectrum of a QD at resonance (solid curve) and for negative detuning (dashed curve) of $\Delta = -40$ μeV. The y-axis in both the plots are in dimensionless units and in (b) is multiplied by a factor of 10. We have considered parameters similar to that achievable in experiments: $\Omega = 80$ μeV, $\Delta_p = 20$ μeV, $\Gamma_0 = \Gamma_\pm = \gamma = 0.84$ μeV, $\Gamma_p = 0.34$ μeV, $\Gamma_d = 3.84$ μeV and $T = 6$K.

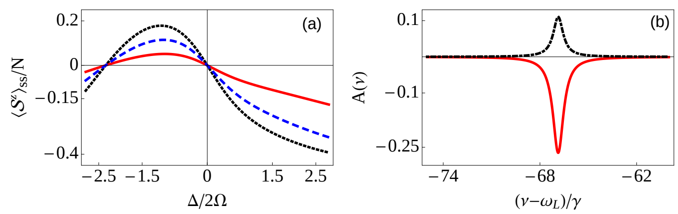

FIG. 3: (color online) (a) Steady-state population inversion in the bare-state for $N = 10$ (dotted curve), $N = 5$ (dashed line) and $N = 1$ (solid curve) QDs. (b) Absorption spectrum of a single QD for $\Delta/2\Omega = -1$ (maximum inversion in Fig. 3a) in presence (solid curve) and absence (dot-dashed line) of phonons. In (b) the y-axis is in dimensionless units and multiplied by a factor of 100 while the x-axis by 0.01. Parameters for the plots are: $\Omega = 2$ meV, $\Delta_p = 20$ μeV, $\Gamma_0 = \Gamma_\pm = \gamma = 0.84$ μeV, $\Gamma_p = 20\gamma$, $\Gamma_d = 40\gamma$ and $T = 6$K.

uated by inserting Eq. (7) in the DME and taking the steady-state $d\rho/dt = 0$ condition. On doing so we obtain,

$$\eta = \ln\left(\chi_+/\chi_-\right)/2. \tag{8}$$

We next look for the inversion in the bare-states. In the secular approximation the collective bare-state inversion is given by,

$$\langle \mathcal{S}^z \rangle_{ss} = \left(\Delta/2\bar{\Omega}\right) \langle \mathcal{R}^z \rangle_{ss}/2, \tag{9}$$

where $\langle \mathcal{R}^z \rangle_{ss}$ is the collective steady-state inversion in the dressed-state found from Eqs (7) and (8) as [6]:

$$\langle \mathcal{R}^z \rangle_{ss} = N\left(\frac{1+e^{2\eta(N+1)}}{1-e^{2\eta(N+1)}}\right) + \frac{2(e^{2\eta(N+1)} - e^{2\eta})}{(1-e^{2\eta})(1-e^{2\eta(N+1)})}. \tag{10}$$

To gain physical insight into the behavior of the bare-state population we need to understand the inversion of dressed-state populations. For this purpose we consider two limits of Eq. (8): (i) $e^{2\eta} = \xi = \chi_+/\chi_- \ll 1$ and (ii) $\xi = \chi_+/\chi_- \gg 1$. From Eq. (10), we get in the limit (i): $\langle \mathcal{R}^z \rangle_{ss}/N \simeq 1 - [2\xi/N(1-\xi)]$, which clearly approaches maximum inversion $\langle \mathcal{R}^z \rangle_{ss}/N = 1$, for $N \gg 1$. As N represent the number of QDs in the ensemble, we can conclude that collectivity among the QDs enhances population inversion in the dressed-state. In the limit (ii) on the other hand we get: $\langle \mathcal{R}^z \rangle_{ss}/N \simeq [2/N(\xi-1)] - 1$ which for $N \gg 1$ becomes, $\langle \mathcal{R}^z \rangle_{ss}/N = -1$, suggesting that all the population tends to reside in the lower dressed-state $|-\rangle$ and, thus, no inversion. Note that, in the limit $\chi_+/\chi_- \to 1$, $\langle \mathcal{R}^z \rangle_{ss}/N \to 0$ there is also no inversion as the dressed-states are equally populated. Additionally, we would like to emphasize that inversion is possible *only in presence of the radiative decay* $\Gamma_\pm$ *among the dressed-states* with $\Gamma_\pm > \Gamma_p$. This can be proved by analyzing the expressions for $\chi_\pm$. On substituting $\Gamma_\pm = 0$ we have $\chi_+/\chi_- = [(\bar{n}+1)\Gamma_p + \Gamma_d]/[\bar{n}\Gamma_p + \Gamma_d] \geq 1$, for any bath temperature and QD parameters thereby implying *no possible dressed-state inversion*.

It is clear from the above discussion and Eqs (8) and (9), that for conditions of inversion in the dressed-state there will be no population inversion among the bare-states $|1\rangle$ and $|2\rangle$. However, at low temperature (small $\bar{n}$), if $\Gamma_p \gg \Gamma_\pm$ we can satisfy the condition $\chi_+/\chi_- \gg 1$ and $\langle \mathcal{R}^z \rangle/N < 0$ thereby leading to inversion among the bare-states for negative laser detuning.

In Fig. 3(a), we show the steady-state population inversion among the bare-states for different number of QDs and typical experimental parameters [27]. Inversion is seen to be achievable for certain range of negative laser detuning as expected from the above discussion. Additionally, we find collectivity induced enhancement of population inversion in the bare-state. This follows directly from the behavior of $\langle \mathcal{R}^z \rangle_{ss}/N$ for $\chi_+/\chi_- \gg 1$. We report almost 40% inversion for reasonable ($N = 10$) number of QDs (complete inversion in bare-state corresponds to a $\langle S^z \rangle/N = 0.5$). Note that the mean-phonon number depends on the generalized Rabi frequency and, thus, varies as detuning varies. This has been explicitly considered for the plots and we have checked that the dependence does not affect our findings and conclusions for the domain of parameters used.

In Fig. 3(b) we show the behavior of the imaginary part of photon absorption spectrum of Eq. (5) at maximum bare-state inversion for a single QD. The spectrum show completely new feature in presence of phonons - *gain at bare-state inversion*. However, for $\Gamma_\pm \gg \Gamma_p$ or in absence of phonons, we get back the typical absorption peak instead of gain (dot-dashed line in Fig. 3b). This implies that we can get amplification of a probe laser from a QD ensemble.

Applications: For a small ensemble of QDs in a microcavity, we find that collectivity enhances the inversion substantially both in bare and dressed-state. Hence, the *phonon induced* gain characteristics of photon-absorption spectrum that we report here for bare-state inversion, can be utilized towards amplification of a weak light field at optical frequencies in such solid-state system. The *photon induced* gain characteristic of the phonon-absorption spectrum for dressed-state inversion, on the other hand, can amplify a phonon wave at THz frequencies tuned to

the $2\bar{\Omega}$ transition (as $\bar{\Omega} \sim$ ps is achievable in QDs).

In conclusion, we have derived a collective dressed-state master equation for an ensemble of strongly driven QDs coupled to phonon and vacuum reservoirs. As a key finding, we reported gain characteristic in the photon and phonon absorption spectrum instead of transparency and absorption respectively due to crucial interplay of the vacuum and phonon reservoirs. Furthermore, we have analyzed the conditions for achieving population inversion in the dressed and bare-states. We have shown that collectivity among the QDs can enhance the obtained population inversion. Finally, as applications, we have proposed amplification of phonon waves at THz frequencies and light fields at optical frequencies. Our findings show the possibility for amplification of photon and phonon waves in QDs and can open new directions towards creation of nanoscale optical and acoustic coherent sources.

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